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Investigation of Electric Field Distribution of A Transformer Using Moving Finite Element Method

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Abstract - The one of the commonly used methods for solution of partial differential equations is the finite element method. Solution area for the differential equation to be solved in this method; are divided into a number of sub-regions called simple , small , interconnected , finite elements.However , especially in time-dependent partial differential equations, analysis is performed using the moving finite element method instead of the classical finite element method where the solution network changes locally; both faster and more accurate.In this work, moving finite element method is considered. The details of how the original variation on the solution network for he moving finite element method and the monitor function selection, which is an important factor in these changes, are detailed in the two-dimensional case. As application, C ++ based software is implemented and analyzed the transformer's electric area distribution according to the state of the classical and moving finite elements and the results are compared.

Index terms - Mesh Generation , Finite Element Method, Moving Finite Element

I. Introduction

In order to solve a large number of problems in engineering and physical sciences, it is necessary to formulate these problems as mathematical expressions and then find out the functions forming the solutions of the problems by using some boundary conditions related to these problems and the initial conditions. These mathematical expressions that formulate a known problem sometimes include derivatives of at least first order or higher order of the sought function. This kind of mathematical expression is called a differential equation. Differential equations, especially first- and second-order linear differential equations, are of great importance for theoretical and practical maintenance and find a wide application in all science and engineering branches. The electric field distribution of the transformer is known as partial differential equations [1]. When the electric field distribution of the transformer is analyzed, it is necessary to solve the partial differential equations correctly and quickly. This is usually done with the finite element method. In particular, using the moving finite element method to change the area of the iron area where the electric field distribution of the transformer changes more rapidly will provide a more accurate and faster solution.

II. Finite Elements Method

Most scientists working with the physical environment are faced with two important issues. These are the mathematical formulation of the physical environment and the other is the numerical analysis of the mathematical model. Removal of the mathematical model of the physical environment often requires basic knowledge of mathematics and physics. Most expressions in mathematical formulas are of the type of differential equation related to the system. The solution of these differential equations gives information about the behavior of the working physical system [2].

For most problems it is not difficult to derive the relevant equation. But it is not always possible to obtain exact solutions of these equations. In this case, instead of trying to find a complete solution, we try to solve with a certain error by using approximation methods. Among these are the finite differences and variational methods (eg Rayleigh-Ritz and Galerkin methods) are the most widely used methods in the literature. In the finite difference approach of the differential equation, the subsequent derivatives are represented by the difference sections. Thus, by forcing the boundary conditions of the given physical medium, the obtained algebraic equation is solved and the result is obtained. In the case of variational methods, the equation is transformed into a weighted integral shape, and then it is assumed that the approximate solution on the dominant line is fused. The choice of coefficients to be selected for the solution must be chosen to match the coefficients in the original differential equation [3] . Variational methods include Rayleigh-Ritz method, Galerkin method, Least-Squares method, which are different from each other in terms of integral shape and weight function which are widely used. These approaches have some drawbacks in terms of creating a time-approximation function when working on an arbitrary domain [4].

The finite element method reduces the disadvantages of the conventional variational methods mentioned above. The method uses the following steps for a simple problem:

a. Creation of a finite element solution network of the given region:
b. Derivation of shape functions for all element types on the solution network
c. Appropriate boundary conditions of the system are determined.
d. Finding element attributes
e. Merge element properties:
f. Solution of the equation system, making other calculations as desired.

The solution results of the system of equations can be used to calculate some important parameters of the system. As an example, the solution results for the Laplace equation giving the examination of electric fields give the potential distribution of the system. If desired, calculations such as field strength, stored energy and capacity can be made using the potential values of the nodes [5]. The main features of the finite element method superior to other numerical methods can be listed as follows:

a) The geometry of a given object can be expressed exactly because of the size and shape of the finite elements used.

b) Areas with one or more holes or corners can be easily examined.

c) Objects with different material and geometric properties may be examined.

d) The boundary conditions can be applied easily.

III. Moving Finite Elements Method

One of the most important issues in the solution of partial differential equations is adaptive production of solution net or grid. In a two-dimensional spatial case, a variational approach is widely used for solution network generation and adaptation. The variational approach is a part of mathematical analysis. Mathematical and physics problems and many engineering problems are solved.

Three methods are used to solve time dependent partial differential equations using adaptive techniques. These methods are as follows [6] :

h-refinement: It adds extra nodes to the existing solution network to improve the regional grid resolution.

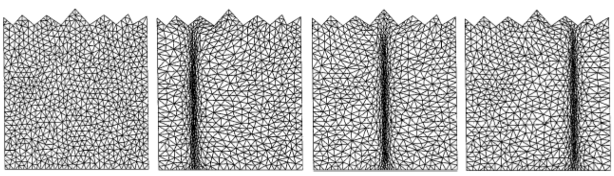
p-refinement: Depending on the accuracy of the solutions, the solution is obtained by choosing the appropriate number of each element used in the end element approach.

r-refinement: Also known as mobile solution network methods. It allows repositioning of the solution network points that are concentrated in the desired region according to the solved differential equation. To do this, it first checks the existing node numbers, then relocates them on the working region as appropriate for the solution.

The p- smoothing method is less popular among the finite elements community. The main reasons for this are the lack of a general formula for achieving reliability, lack of productivity, and solution network movement. The r method has advantages over the h and p methods. In the r method, the solution network changes continuously. This makes it easier to add a time derivation to the differential equation we use. The data structure of the program is easy and easy to implement. In this point, the r-correction technique is divided into static and dynamic methods:

Static methods: The approximate solution is determined on a network initially provided. Throughout the calculation, a new network of solutions (whether or not having the same number of nodes) is produced using existing grid generation techniques. The solution is then interpolated onto the new solution network (intermediate value is found). Thus, redistribution or addition of grid points and interpolation occur at a fixed time in the solution process. Despite success, these methods require large amounts of computational load. Because these changes in the solution network require the codes of the program to be rewritten.

Dynamic methods: Also known as mobile solution networking methods; node speeds are added to solution network equality. Densification is achieved in regions where the nodes change rapidly depending on the time of the solution. Usually, the number of nodes in the grid remains the same in these methods. For this type of mobile solution method, it is necessary to consider two combined equations. These equations are moving solution network equation (controlling the change of the solution network) and the original differential equation of the problem. The solution network is constantly changed without the need for any interpolation step. Solution network equality and original differential equality are usually solved simultaneously.



(a) (b) (c) (d)

Fig 1. Technique for producing moving grid [7]

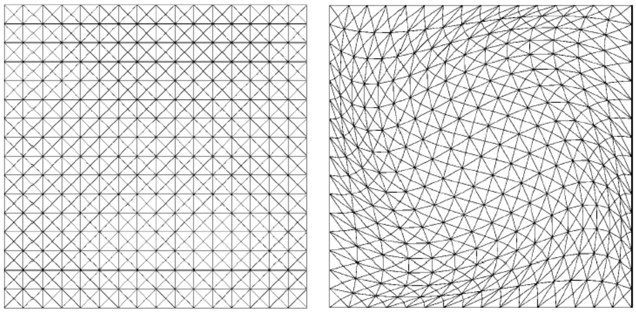


Fig 2. Global relocation of nodes on the solution network

As can be seen from *Figs. 1 and 2*, the technique of producing a moving grid is to achieve a controlled condensation in a certain region of interest. This density operation is performed by a function known as a monitor function. The Monitor function is a function that the user specifies. With this function, the solution network density is checked. This control is achieved by selecting the appropriate monitor function to achieve successful and accurate results [8] .

IV. Application

A. Calculation of Magnetic Size of a Transformer By Finite Elements Method

In this study, the magnetic quantities of a transformer shown in *Fig. 3* are calculated by the finite element method. Due to the symmetry in this transformer, a quadratic part of the transformer is considered sufficient, and the results obtained are multiplied by four to obtain the complete analysis results of the transformer.

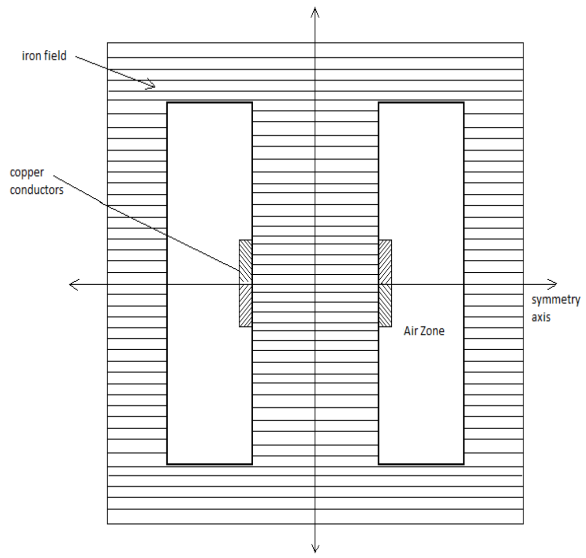


Fig 3. Transformer studied by finite elements method

a. Determination of Solution Region

In order to solve this problem, it is necessary to first determine the region where the vector potential value is zero except for the transformer. In this study, it is assumed that the potential value is zero at the boundary where the divider is made. However, it is also true that the potential value there is not zero in theory. The farther this limit is taken from the transformer, the closer it approaches the zero potential value. In this case, the results of the solution will be more accurate, and the operations will be prolonged and the solution operations will be difficult. There are also programs that automatically extend these limits. However, this technique has not been utilized in our study, and the effect of this limit on the calculation will be negligible [9].

b. Constitution of The Solution Mesh

One of the oldest problems encountered in computer graphics is the surface model which will be defined by these boundary curves according to the four boundary curves given. Since the transformer is defined according to four boundary curves, the solution network is produced with Coons surface. Let the boundary curves *x(u,0), x(u,1), x(0,v) and x(1,v)* The bilinear Coons patch can be expressed as:

$$x(u,v) = (1-u).x(0,v) + u.x(1,v) + (1-v).x(u,0) + v.x(u,1) - \begin{bmatrix}1-u & u\end{bmatrix} \begin{bmatrix} x(0,0) & x(0,1) \\ x(1,0) & x(1,1) \end{bmatrix} \begin{bmatrix} 1-v \\ v \end{bmatrix} \quad (1)$$

Where *x(0,0), x(0,1), x(1,0) and x(1,1)* are each corner coordinates. Boundary curves are discrete points. Any curve modeling technique (Bezier, B_Spline, etc.) can be used to generate these curves [10].

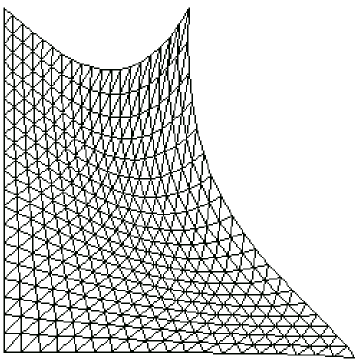


Fig 4. Coons surface

Bezier and B_Spline curves are approximate curves according to the control point given. By developing these curve modeling techniques, the problem of holding and storing all of the boundary curves in the course of forming the Coons surface has been eliminated, so that only the control points are held and the operations are performed accordingly. *Equation (2)* shows the general state of the Bezier curve.

$$P(u) = \sum_{i=0}^{n} B_i \cdot J_{n,i}(u) \quad [2]$$

$J_{n,i}(u)$: The combining function is as *equation (3).*

$$J_{n,i}(u) = \binom{n}{i} \cdot u^i \cdot (1-u)^{n-i} \quad [3]$$

Where n is the number of control points and $B_{i,j}$ are the control points. Another curve modeling technique used to obtain boundary curves is B_Spline curves. *Equation (4)* shows the general state of the B_Spline curve. n is the control point number, $B_{i,j}$ is the control point and k is the number of the curve to be generated [11].

$$P(u) = \sum_{i=0}^{n} B_i \cdot N_{i,k}(u) \quad ,$$

$$N_{i,k}(u) = \begin{cases} 1 & x_i \le u \le x_{i+1} \text{..için} \\ 0 & \text{diğ} \end{cases} \quad [4]$$

$$N_{i,k}(u) = \frac{(u-x_i)\cdot N_{i,k-1}(u)}{x_{i+k-1}-x_i} + \frac{(x_{i+k}-u)\cdot N_{i+1,k-1}(u)}{x_{i+k}-x_{i+1}} \quad [5]$$

Figure 5 shows Bezier and B_Spline sample curves.

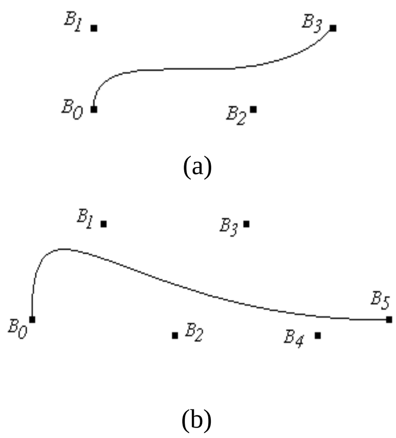


Fig 5. (a) Bezier curve (b) B_spline curve

c. Obtaining the Solution

The coordinates of all triangular elements of the transformer, the corner node numbers, the coordinates of the nodes forming each triangular element and the region in which the element is located are written in a file and the *S* values multiplied by the current density *J* according to the region in which the elements are located are written in a format suitable for another file. After these files are created, the appropriate *S* values are calculated according to the region for each triangular element. Then, these *S* values are substituted in the *[ST]* matrix as previously described to obtain this matrix. For nodes with current density, these values are assigned in the *[RHS]* matrix. *[ST] [A] = [RHS]* matrices are obtained by writing these values of *[ST]* and *[RHS]* for the nodes with zero potential values [12]. By using Gauss elimination method, this matrix is solved and vector potential values are obtained. For the transformer, the current density is taken as ***J = 4000000*** **A/cm^2** and the solution is calculated as ***E = 4,42432 Joule*** and inductance ***L = 9,0223.10^{-2} Henry***. Appropriate equipotential curves are obtained by using these vector potential values.

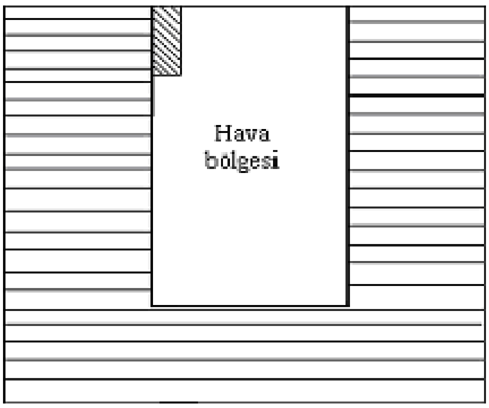


Figure 6. One-quarter of the transformer

B. Calculation of Magnetic Quantities of Transformer by Moving Finite Elements Method

When calculating the magnetic quantities of the transformer by means of moving finite elements, the point to be considered is the selection of the appropriate monitor function. With the help of this convenient monitor function, the control of the solution network is ensured and the concentration of the solution network in the region where the solution changes rapidly; namely more triangular elements are used. The number of elements and nodes on the solution network is kept constant, but a faster solution is obtained by providing a concentration to the region where the solution changes rapidly. Values close to the results obtained with the finite element method commonly used in the calculations made are obtained. The only difference there is; the size of the matrix is smaller than that of the main matrix used in the analysis of the classical finite element; in other words, fewer elements. Because only transactions were done on the regions where the solution was changed quickly, fewer datas were needed.

C. Structure of the Developed Program and Result

As a practical application, a C ++ based software was implemented to analyze both the conventional finite element and the moving finite element of the transformer given in Figure 3. The results of the program are as follows.

a. Classical Finite Element Analysis Program:

The part of the program that does the classic finite element analysis is as follows.

```
------------ TRAFO ANALIZ PROGRAMI --------------------------
-------------------------------------------------------------
------- Düğüm Numarası ve koordiatları ==============>1 ----
------- Eleman Numarası ve Düğüm Numarası   =========>2 ----
------- A Vektör Potansiyel Değerleri için =========> 3 ----
------- B Manyetik Akı Yoğunluğu leri için =========> 4 ----
------------------- MESH CIZIMI     ================> 5 ----
-------------------- EGRILER   =====================> 6 ----
----------- CIKIS -----   =========================> 7 ----
```

Fig 7. Program structure that performs analysis of classical finite element

The results of the program according to the order of operation are as follows:

When the program is run, the menu shown in *Figure 7* is displayed and the digits *1* to *6* are pressed to see the desired values or figures.

When button *1* is pressed, the numbers and coordinates of the nodes used in the finite element analysis of the transformer are calculated.

When the button *2* is pressed, the number of the elements and the number of nodes each of these elements are made of.

When button *3* is pressed, vector potential values are calculated which are calculated after the main matrix *[S]* is generated and solved.

When the button *4* is pressed, the magnetic flux density values *B* are obtained by using this calculated vector potential.

When button *5* is pressed, the solution network required for classical end element analysis is seen. *(Figure 9)*

When the button *6* is pressed, the equipotential curvature of the transformer is shown by taking advantage of the vector potential value (*Figure 10, Figure 11*)

b. Moving Finite Element Analysis Program:

When analyzing the transformer by moving finite element method, firstly the regions where the solution changed quickly were determined. As is known, in a transformer, the magnetic field in the iron region changes rapidly. For this reason, the region where the transformer is changing rapidly is divided into more triangular elements so that the operation is more accurate. With the aid of a suitable monitor function, only the iron part of the transformer is divided into more elements, and the results obtained in this case are given at other stages of the program. The monitor function used here is determined by trial and error. Since the transformer is defined according to four boundary curves, the selected monitor function must be selected accordingly. The selected monitor function in the program is shown in equation (6).

$$M = \sqrt{1 + (x * y)^{\frac{2}{3}} t} \qquad [6]$$

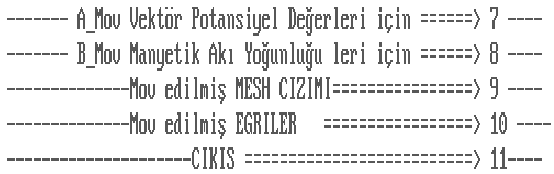


Fig 8. Program structure for moving finite element analysis

Figure 8 shows the part of the developed program that analyzes the moving finite elements. The results of the program according to the order of operation are as follows:

When button *7* is pressed, A vector potential values are calculated using the moving finite element analysis of the transformer.

When the button *8* is pressed, the values of the magnetic flux density *B* are calculated by using the vector potential values obtained.

When button *9* is pressed, the solution network required for moving finite element analysis is shown (*Figure 12*).

When button *10* is pressed, the calculated equipotential curves are plotted with the help of vector potential values. (*Figure 13*)

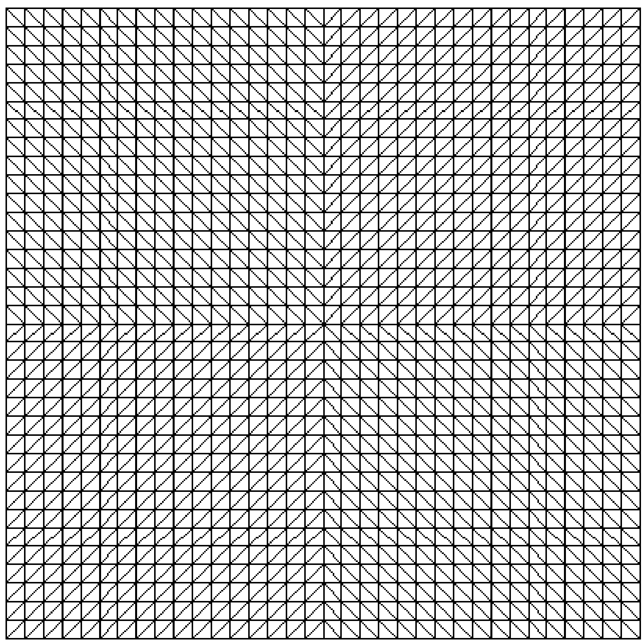


Fig 9. Solution mesh production of transformer

After the mesh is generated, the equipotential curves of the transformer are obtained as follows.

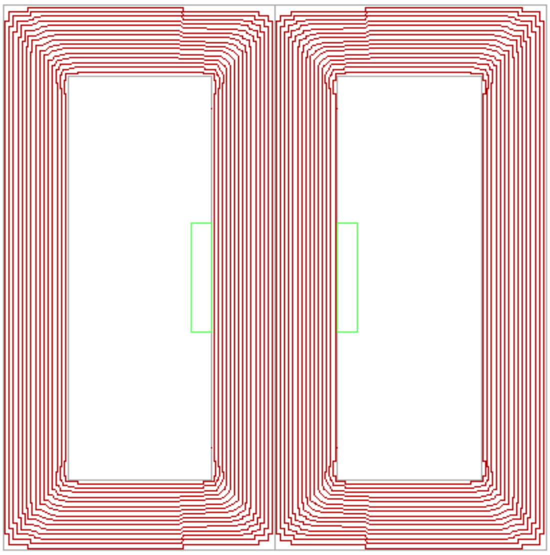


Figure 10. Equipotential curves of the transformer (15 curves)

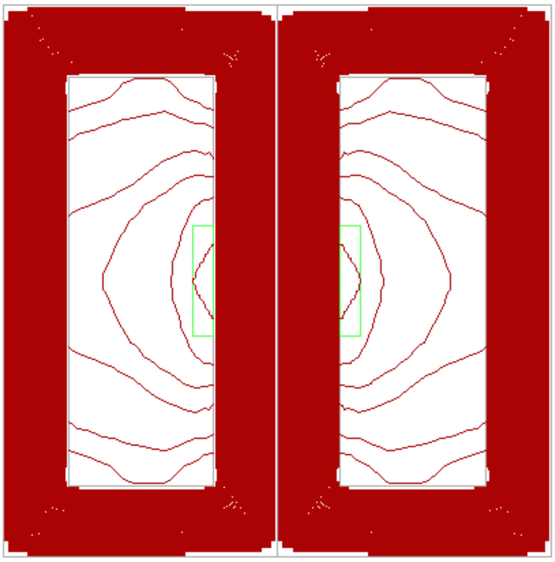


Figure 11. Equipotential curves of the transformer (250 curves)

The solution network required for performing the moving finite element analysis of the transformer is shown in *Figure 12*. The number of nodes used here is 469 and the number of elements is 1122. The equipotential curves drawn with the help of the obtained A vector potential values are shown in *Figure 13*. A comparison of the resulting *A* values of the moving finite element and the classical finite element analysis is given in *Figure 14*.

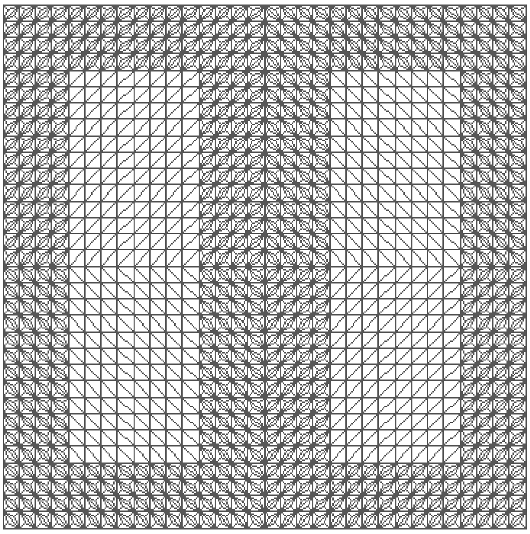


Figure 12. Mesh generation of transformer using moving finite element

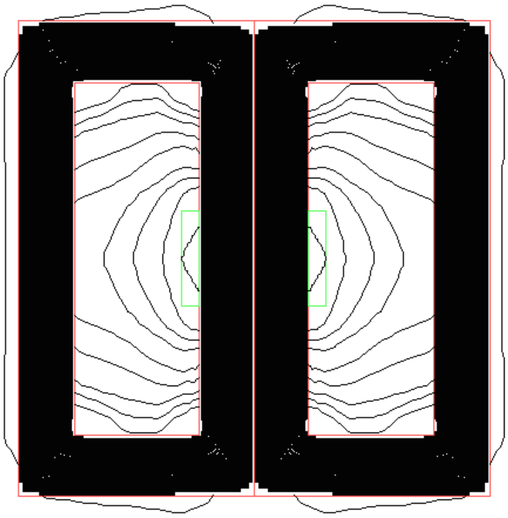


Figure 13. Equipotential curves of the transformer (250 curves)

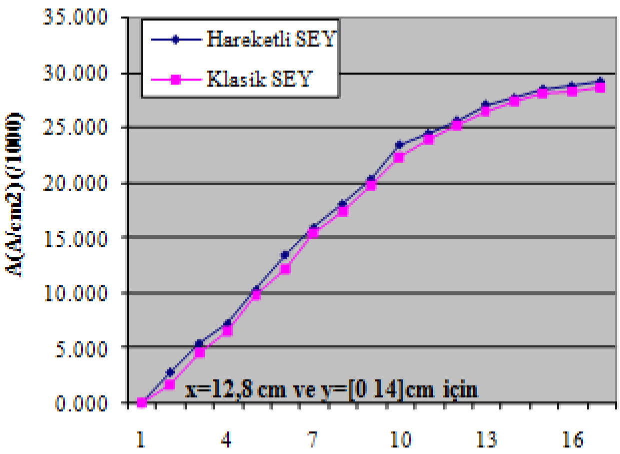


Fig 14. A magnetic vector potential values for the transformer x = 12,8 and y = [0 14] cm

V. CONCLUSION

In this study, mainly the method of moving finite element and the selection and use of monitor function required for this method are given. With the help of the developed program, a classical and moving finite element analysis of a transformer is made. The vector potential values obtained from both analyzes were compared and the equipotential curves were plotted. When moving finite element analysis is performed, faster and more accurate solution is achieved by increasing the number of mesh generation elements in the iron zone where the magnetic flux changes rapidly.

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